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Combining and comparing astrometric data from different epochs: A case study with Hipparcos and Nano-JASMINE

Daniel Michalik¹, Lennart Lindegren¹, David Hobbs¹, Uwe Lammers², and
Yoshiyuki Yamada³

¹*Lund Observatory, Lund University, Box 43, SE-22100 Lund, Sweden
e-mail: daniel.michalik, lennart, david@astro.lu.se*

²*European Space Agency (ESA/ESAC), P.O. Box 78, ES-28691 Villanueva de la
Cañada, Madrid, Spain, e-mail: uwe.lammers@sciops.esa.int*

³*Department of Physics, Kyoto University, Oiwake-cho Kita-Shirakaw
Sakyo-ku, Kyoto, 606-8502 Japan, e-mail: yamada@amesh.org*

Abstract. The Hipparcos mission (1989-1993) resulted in the first space-based stellar catalogue including measurements of positions, parallaxes and annual proper motions accurate to about one milli-arcsecond. More space astrometry missions will follow in the near future. The ultra-small Japanese mission Nano-JASMINE (launch in late 2013) will determine positions and annual proper motions with some milli-arcsecond accuracy. In mid 2013 the next-generation ESA mission Gaia will deliver some tens of micro-arcsecond accurate astrometric parameters. Until the final Gaia catalogue is published in early 2020 the best way of improving proper motion values is the combination of positions from different missions separated by long time intervals. Rather than comparing positions from separately reduced catalogues, we propose an optimal method to combine the information from the different data sets by making a joint astrometric solution. This allows to obtain good results even when each data set alone is insufficient for an accurate reduction. We demonstrate our method by combining Hipparcos and simulated Nano-JASMINE data in a joint solution. We show a significant improvement over the conventional catalogue combination.

1. Introduction

Stellar proper motions have traditionally been computed by comparing positional catalogues based on observations made at different epochs (typically separated by several decades). Parallaxes were either ignored in this process, or determined by quite different instruments and methods. With the advent of Hipparcos and space astrometry, it has become necessary to treat the determination of positions, proper motions, and parallaxes in a unified manner, i.e., in a single least-squares solution. This was the principle used for the construction of the Hipparcos and Tycho catalogues (ESA 1997), as well as for the new reduction of the Hipparcos data (van Leeuwen 2007), and it will be used for the Gaia mission and other planned space astrometry projects. In the future we will therefore have access to several independent astrometric catalogues, one for each space project. Improved proper motions can again be computed by comparing the positions in catalogues at different epochs. However rather than combining the results of the

catalogues in the conventional manner, we propose to make a *joint solution* of the data from the missions. In this paper we explore possible advantages of this approach, combining the Hipparcos Catalogue and simulated data from the Japanese Nano-JASMINE mission as a study case.

Nano-JASMINE (launch planned for late 2013) is an ultra-small Japanese satellite to measure the astrometric parameters of about one million celestial sources to 12th magnitude. The expected accuracy of the positions, parallaxes and annual proper motions of magnitude 7.5 objects is about 3 mas. Nano-JASMINE is a very small technological demonstrator for bigger follow-up missions. Its data will be reduced using the Astrometric Global Iterative Solution (AGIS) being developed for the Gaia mission at ESA/ESAC and Lund Observatory (Lindegren et al. 2011).

2. Theory

The estimation of stellar astrometric parameters from observational data can be cast as a linear least-squares problem. The optimum estimate of the unknowns \mathbf{x} is obtained by solving the normal equations $N\mathbf{x} = \mathbf{b}$ where the covariance of \mathbf{x} is given by N^{-1} and where \mathbf{b} are the residuals. In the conventional approach of combining two astrometric catalogues the least-squares solution of both data sets is done independently and the combination is done a posteriori. If σ_1 and σ_2 are the accuracies in the two catalogues, positions and parallaxes improve as $\sigma^{-2} = \sigma_1^{-2} + \sigma_2^{-2}$ by computing weighted means of the values of the two catalogues. Proper motions improve as $\sigma_{pm} = (\sigma_{pos1}^2 + \sigma_{pos2}^2)^{1/2} / \Delta T$ by taking the position difference divided by the epoch difference ΔT . Instead we propose to combine the normal equations of both missions *before* solving

$$(N_1 + N_2)\mathbf{x} = \mathbf{b}_1 + \mathbf{b}_2 \quad \rightarrow \quad N\mathbf{x} = \mathbf{b}, \quad (1)$$

allowing us to retrieve directly $\hat{\mathbf{x}}_{joint}$ of the combined catalogues. This can also be understood in terms of Bayesian estimation (assuming multivariate Gaussian parameter errors), with N_1, \mathbf{b}_1 presenting the prior information, N_2, \mathbf{b}_2 the new data, and N, \mathbf{b} the posterior information. Even if each group is not solvable on its own, the combined data may be solvable.

3. Simulations

Simulations are carried out using AGISLab, a software package aiding the development of algorithms for the data reduction of Gaia, developed at Lund Observatory. For simulations the Hipparcos catalogue is used to generate observations. Three catalogues are created (see Figure 1):

The Hipparcos catalogue contains the source parameters read from the Hipparcos catalogue file including their covariance matrix. The Hipparcos data are available for the reference epoch J1991.25. To combine them with simulated Nano-JASMINE data the source parameters and covariance matrix have to be propagated to the mid mission reference epoch (which for Nano-JASMINE is expected to be J2015).

The noisy catalogue contains the starting values for the data processing. It is created by perturbing the original sources with random noise of a given amplitude.

The simulated “true” catalogue defines the sources used for creating the simulated true observations (which may be perturbed subsequently). For the real mission

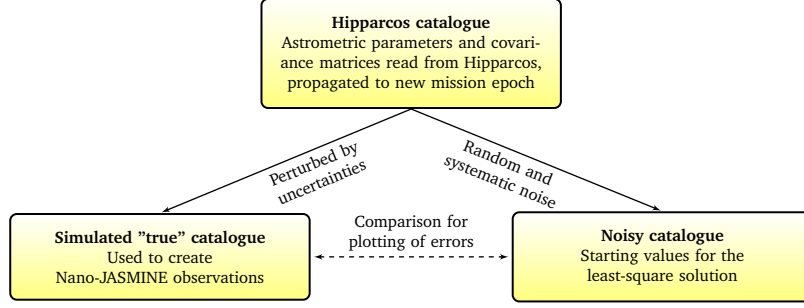


Figure 1. Relationships between catalogues

the true catalogue is not known. The Hipparcos sources give the observed astrometric parameters and their uncertainties. The true parameters of the sources are expected to be at an unknown position within the error space of the original sources. The five astrometric parameters of the simulated true catalogue require five independent Gaussian variates scaled by the square root of the covariance matrix \mathbf{C} . To find a unique square root we use the lower triangular matrix \mathbf{L} resulting from the Cholesky decomposition $\mathbf{C} = \mathbf{L}\mathbf{L}^T$. Then $\mathbf{e} = \mathbf{L}\mathbf{g}$, where \mathbf{g} is a vector of five independent unit Gaussian values and \mathbf{e} is the resulting vector of errors to be applied to the astrometric parameters. Since $E(\mathbf{g}\mathbf{g}^T) = \mathbf{I}$, where $E(\dots)$ denotes the expectation value and \mathbf{I} is the identity matrix, it is readily verified that \mathbf{e} has the desired covariance $E(\mathbf{e}\mathbf{e}^T) = \mathbf{C}$.

In order to make a joint solution the Hipparcos normal equations are reconstructed from the covariance inverse, $N_{HIP} = \mathbf{C}_{HIP}^{-1}$. The right-hand side of the Hipparcos normal equations needs to be chosen such that solving for the Hipparcos information only, the update would bring the current parameter values back to the original Hipparcos values. This can be done by calculating a vector \mathbf{d} defined as the difference between the original Hipparcos source parameters (subscript o) and the current values (subscript c) and choosing the right-hand side \mathbf{b}_{HIP} as

$$\mathbf{b}_{HIP} = N_{HIP}\mathbf{d} = \mathbf{C}_{HIP}^{-1} \begin{pmatrix} (\alpha_o - \alpha_c) \cos \delta_o \\ \delta_o - \delta_c \\ \pi_o - \pi_c \\ \mu_{\alpha\star o} - \mu_{\alpha\star c} \\ \mu_{\delta o} - \mu_{\delta c} \end{pmatrix}. \quad (2)$$

Solving $N_{HIP}\mathbf{x} = \mathbf{b}_{HIP}$ gives $\mathbf{x} = \mathbf{d}$ and the application of this update to the current parameters obviously recovers the Hipparcos parameters.

4. Results

Table 1 shows results of simulation runs. As expected the combination of Hipparcos and Nano-JASMINE gives a great improvement in proper motions. Additionally we show that our proposed joint solution performs significantly better than the conventional catalogue combination method. This can be understood as follows. The astrometric parameters in the Hipparcos (or Nano-JASMINE) catalogue are correlated. The large improvement of the proper motions therefore brings some improvement also to the

other parameters, provided that the correlations are properly taken into account. This is the case for the joint solution, but not for the conventional combination.

	Position @J2015 [mas]		Parallax [mas]	Proper motions [mas/year]	
	α	δ	π	$\mu_{\alpha\star}$	μ_{δ}
mag < 7.5, ~ 15 000 stars					
Hipparcos only (Hip)	18.19	14.84	0.80	0.77	0.63
Nano-JASMINE only (NJ)	2.56	2.54	3.05	4.65	4.50
Conventional combination Hip + NJ	2.54	2.51	0.77	0.111	0.110
Joint solution Hip + NJ	2.41	2.40	0.75	0.108	0.105
Improvement of joint solution	5.2%	4.4%	3.5%	3.2%	4.4%
mag < 11.5, ~ 117 000 stars					
Hipparcos only (Hip)	27.06	22.35	1.18	1.14	0.94
Nano-JASMINE only (NJ)	4.57	4.53	5.43	8.38	8.02
Conventional combination Hip + NJ	4.51	4.44	1.15	0.197	0.194
Joint solution Hip + NJ	4.43	4.26	1.11	0.188	0.185
Improvement of joint solution	1.8%	3.9%	4.0%	4.5%	4.5%

Table 1. Conventional catalogue combination vs. a joint astrometric solution, for a subset of bright stars and for all Hipparcos stars. Simulations of Nano-JASMINE are based on a conservative observation performance model and an optimal scanning law. The positions from Hipparcos have been propagated to the Nano-JASMINE mid-mission epoch J2015.

4.1. Future work

We are planning to extend our studies of catalogue combination by simulating the improvements that can be gained by applying our method to catalogues from simulated Gaia and Nano-JASMINE data together with the Hipparcos and the Tycho-2 catalogues.

Furthermore, the goodness-of-fit of the combined solution is sensitive to small deviations of the stellar motions from the assumed (rectilinear) model. We are investigating how this can be used to identify binary candidates with orbital periods of decades to centuries. This will contribute to the census of the binary population within a few hundred parsecs from the sun by filling a difficult-to-observe gap between the shorter period spectroscopic binaries and the longer period visually resolved systems.

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